

CHAPTER-2

DIMENSIONAL ANALYSIS, SIMILITUDE AND HYDRAULIC MODEL

Introduction

Dear students, in this unit you will learn about the basic concepts of dimensional analysis, similitude and hydraulic models. The unit divided into two sections. In the first section you will introduce to the principle of dimensional homogeneity and its application. In the second section you will learn about similitude and hydraulic models.

Objectives

On completion of this unit you are expected to:

- ☞ Understand the concept and application of dimensional homogeneity of dimensional homogeneity
- ☞ Understand dimensional analysis and know its uses.
- ☞ Be capable of solve and prove equations using the two methods of dimensional analysis.
- ☞ Know and explain the different types of hydraulic similarities.
- ☞ Classify and explain the different types of hydraulic models.

SECTION-1: DIMENSIONAL HOMOGENEITY

Definition:

What are dimensions?

Physical quantities can be

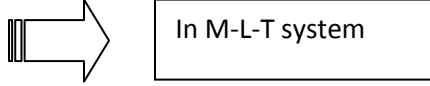
☞ Fundamental quantities

- Mass
- Length
- Time
- Temperature (Only for compressible fluids)

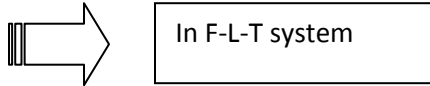
☞ **Derived quantities** this are quantities which are expressed in terms of fundamental quantities.

The dimension of any physical quantities can be expressed as,

☞ **Fundamental dimension**

- Mass [M]
 - Length [L]
 - Time [T]
 - Temperature [θ] (Only for compressible fluids)
- 

OR

- Force [F]
 - Length [L]
 - Time [T]
 - Temperature [θ] (Only for compressible fluids)
- 

The two systems are related by the Newton's Second Law of Motion:

$$F = m \cdot a$$

Dimensionally,

$$[F] = [M] * \left[\frac{L}{T^2} \right]$$

$$[M] = \left[\frac{FT^2}{L} \right]$$

☞ **Derived dimensions** these are dimensions which are expressed in terms of Fundamental dimension. For example, velocity is defined as the distance moved per unit time and therefore its dimensions can be derived as

$$velocity = \frac{length}{time} = \left[\frac{L}{T} \right]$$

Likewise, the dimensions of other physical quantities may be obtained. Table 2.1 gives the dimensions of various physical quantities used in mechanics in both systems.

Table 1.1: Dimensions of various physical quantities

S.No.	Quantity	Symbol	Dimensions	
			M-L-T	F-L-T
1	Length	L	L	L
2	Time	T	T	T
3	Mass	m	M	M
4	Force	F	$M L T^{-2}$	F
5	Velocity	V	$L T^{-1}$	$L T^{-1}$
6	Acceleration	a	$L T^{-2}$	$L T^{-2}$
7	Area	A	L^2	L^2
8	Discharge	Q	$L^3 T^{-1}$	$L^3 T^{-1}$
9	Pressure	P	$M L^{-1} T^{-2}$	FL^{-2}
10	Power	P	$ML^2 T^{-3}$	FLT^{-1}
11	Density	ρ	$M L^{-3}$	$FT^2 L^{-4}$
12	Unit gravity force	γ	$ML^{-2} T^{-2}$	FL^3
13	Dynamic viscosity	μ	$M L^{-1} T^{-1}$	FTL^2
14	Kinematic viscosity	ν	$L^2 T^{-1}$	$L^2 T^{-1}$
15	Surface Tension	σ	MT^{-2}	FL^{-1}
16	Bulk modulus of elasticity	K	$ML^{-1} T^{-2}$	FL^{-2}

Activity 2.1

State and define the two types of dimensions

2.1 Dimensional homogeneity

A physical equation is said to be dimensionally homogeneous if the quantities on both sides of the equation have identical dimension.

A dimensionally homogeneous equation is applicable to all system of units. On the other hand, a dimensionally non homogeneous equation is applicable only to the system of units for which it had derived. Let us consider the following two equations:

A) $p = \gamma h$

B) $v = \frac{1}{N} R^{2/3} S^{1/2}$

Eq. (A) is dimensionally homogeneous. This can be proved by substituting the dimensions of each term.

$$\left[\frac{F}{L^2}\right] = \left[\frac{F}{L^3}\right] \cdot [L] = \left[\frac{F}{L^2}\right]$$

On the other hand, Eq. (B) is dimensionally non-homogeneous substitute the dimensions Left hand side

$$\text{left hand side} = \left[\frac{L}{T}\right]$$

$$\text{right hand side} = \left[L^{\frac{2}{3}}\right] \cdot [1]$$

Note. The quantities which are dimensionless are represented by [1].

As the dimensions of the two sides are not the same, the equation is non-homogeneous. Eq. (B) is only applicable to SI-system. It cannot be used in FPS. In fact, the Manning formula in FPS system is given by

$$V = \frac{1.486}{N} \cdot R^{2/3} \cdot S^{1/2}$$

Activity 2.2

What is the criterion for a given physical equation to be dimensionally homogeneous?

Application of the principle of dimensional homogeneity to determine the dimensions of a physical quantity

(1) To determine the dimensions of a physical quantity.

The dimensions of a physical quantity can be determined using the principle of dimensional homogeneity. Let us now determine the dimensions of the kinetic energy (E) from the equation:

$$E = \frac{1}{2} \cdot mv^2$$

Substitute the dimension of the right-hand side,

$$[E] = \left[\frac{F \cdot T^2}{L}\right] \cdot \left[\frac{L^2}{T^2}\right]$$

$$[E] = [FL]$$

Example 1.1 Find the dimension of the following quantities in both M-L-T and F-L-T system:

- (a) Discharge
- (b) Momentum
- (c) Power
- (d) Torque

(2) To convert units from one system to another system.

The principle of dimensional homogeneity can be used to convert units from one system to the other system. For example, let us convert the pressure from KN/m^2 to $lb/inch^2$. we know

$$1 lb = 453.6 gm$$

Or

$$1 Kg = 2.204 lb$$

And

$$1 inch = 2.54 cm = 0.0254 m$$

Also

$$1 Kg = 9.81 N$$

Or

$$1 N = 0.102 kg = 0.2247 lb$$

$$\begin{aligned} \text{Conversion factor} &= \frac{KN/m^2}{lb/inch^2} = \frac{KN}{lb} * \frac{(inch)^2}{(m)^2} \\ &= 0.2247 * 10^3 * (0.0254)^2 = 0.145 \end{aligned}$$

Therefore, the pressure in KN/m^2 can be converted into $lb/inch^2$ by multiplying it with 0.145. In other words,

$$\text{pressure in } lb/inch^2 = 0.145 * \text{pressure in } KN/m^2$$

Exercise 2.1 show that $1 kg/cm^2 = 98.1 kn/m^2$

(3) To check whether a given equation is Homogeneous and convert it to the other system if non-homogeneous

The principle of dimensional homogeneity can be used to check whether a given equation is dimensionally homogeneous. If the equation is not dimensionally homogeneous, it can be made applicable to the other system by first determining the dimension of the coefficient.

For example, the equation

$$V = \frac{1}{N} \cdot R^{2/3} \cdot S^{1/2} = CR^{2/3} \cdot S^{1/2} (\text{say})$$

As proved before, is dimensionally non-homogeneous.

$$\text{The dimension of the coefficient "C" } = \left[L^{1/3} / T \right]$$

Because,

$$C = \frac{V}{R^{2/3} \cdot S^{1/2}} = \frac{[L/T]}{[L^{2/3}][1]} = \left[L^{1/3} / T \right]$$

Since N is pure constant depending only upon the type of surface, its value is the same in both MKS and FPS (foot-Pound-Second) system. In order to make the above equation applicable to FPS the numerator 1 in the coefficient $\left[\frac{1}{N} \right]$ will be changed. We know $1m = 3.281 feet$. Remembering that the unit of time same in both system.

$$\text{Conversion factor for } C = \left[L^{1/3} / T \right] = (3.821^{1/3}) = 1.486$$

Therefore, the equation in F.P.S. system is

$$V = \frac{1.486}{N} \cdot R^{2/3} \cdot S^{1/2}$$

Exercise 2.2 The following equations are applicable in SI system. Find the dimensions of the constants and their corresponding value in FPS system.

- (a) $50\sqrt{RS}$
- (b) $1.42 \tan \theta / 2 \ H^{5/2}$
- (c) $1.84bH^{35/2}$

(4) Dimensional analysis

Hydraulics is an experimental science & is also a complex subject. Most phenomena involving the movement of water are known to be dependent on many variables- geometric characteristics, fluid properties and flow characteristics. Much of raw experimental data were incomplete, ambiguous, & incorrectly taken.

It is usually impossible to determine all the essential facts for a given fluid flow by pure theory, & hence, dependence must often be placed up on experimental investigations.

The number of tests to be made can be greatly reduced by the systematic use of Dimensional Analysis and the laws of similitude or similarity. Dimensional Analysis is a mathematical technique, which makes use of the study of dimension as an aid to the solution of several engineering problems

Dimension less grouping reduces the number of variables that have to be processed.

Dimension analysis can be used to obtain functional relationship among the variables in terms of non- dimensional parameter. It helps in obtaining a systematic form of the variables involved in the problem. However, dimensional analysis doesn't give complete relationship; it gives only a general relationship.

Dimensional analysis is the method of analysis based on the mathematics of the dimension of quantities.

In dimensional analysis, from a general understanding of fluid phenomenon, one first predicts the physical parameters that will influence the flow, and then by grouping these parameters in dimension combinations, a better understanding of the flow phenomena is made possible.

Uses Of dimensional analysis

- ☞ It can be used to obtain a functional relationship among the variables in terms of non-dimensional parameters.
- ☞ Dimensional analysis reduces the number of experiments required in a particular investigation.
- ☞ Dimensional analysis helps in obtaining a systematic form of the variables involved in a particular fluid phenomenon.
- ☞ It gives a sound and orderly arrangement of the variables involved in the problem.
- ☞ Deriving equations expressed in terms of non- dimensional parameters to show the significance of each parameter.

However, dimensional analysis does not give the complete relationship. It gives only a general qualitative expression. Investigations have to be done to obtain the complete expression. The numerical values of the coefficients are usually obtained from experimental investigations.

The following two methods of dimensional analysis are commonly used:

- (a) Rayleigh's method
- (b) Buckingham's π -theorem.

(a) Rayleigh's method

In this method, the functional relationship is expressed in an exponential form. If Y is some function of independent variables, X_1, X_2, X_3 etc. the function relationship can be written as

$$Y = \Phi[X_1, X_2, X_3, \dots]$$

Where Φ stands for 'a function of', It does not imply anything about the form of the function.

Any function can be expressed as a series of terms each being made up of the product of variables brought to suitable powers. Thus

$$Y = K[X_1^a, X_2^b, X_3^c \dots]$$

In which k is dimension less coefficient which can be determined either from the physical characteristics of the problem or from experiments, The exponents a, b, c etc. are

determine from the principle of dimensional homogeneity, *the exponents of the dimensions on both sides must be same.*

By equating the exponents on both sides, a set of simultaneous equations is obtained. The exponents can be determined by solving these simultaneous equations. Since there are only three fundamental dimensions, only 3 simultaneous equations are found. If the number of exponents involved in the relationship is more than 3, some of the exponents can be expressed in terms of others. The non-dimensional parameters are then formed by grouping the variables with like exponents.

Example 2.2

Obtain an expression for drag force F_D on stationary sphere of diameter D in a fluid of density ρ viscosity μ as a function of non-dimensional group. Let the velocity of the fluid be V

Activity 2.3

Show each steps of solving equations using Rayleigh's method?

Exercise 2.5

The velocity V through an orifice depends upon the diameter 'd' of the orifice, the head H over the crest, the acceleration due to gravity g , the density ρ , the viscosity μ and the surface tension σ . Show that

$$V = C\sqrt{2gH}f\left[\left(d/H\right), \frac{\mu}{\rho g^{1/2}H^{3/2}}, \frac{\sigma}{\rho H^2 g}\right]$$

(b) Buckingham's π –Theorem

The Rayleigh method of dimensional analysis becomes cumbersome when a large number of variables are involved. The Buckingham π –Theorem may be used in such problems.

The Buckingham π –Theorem states that if there are n variables in a dimensionally homogenous equation and if these variables contain m fundamental dimensions (such as, M, L, T), they may be grouped into $(n - m)$ non-dimensional parameters. Buckingham called these non-dimensional parameters as π –terms.

Mathematically, if a variable x_1 depends upon the variable $x_2, x_3 \dots, x_n$, the functional equation may be written as

$$x_1 = [x_2, x_3, \dots, x_n]$$

This equation may be written as

$$f[x_1, x_2, x_3 \dots, x_n] = C$$

Where C is a constant and ' f ' represents some function. In this equation, there are n variables. If there m fundamental dimensions, then according to Buckingham theorem,

$$F = [\pi_1, \pi_2 \dots \pi_{n-m}]$$

Obviously, the number of π -terms is $n-m$.

Each π -term contains m primary variable, which are also called the repeating variables. The repeating variables appear in all π -terms. In addition to these m repeating variables, each π -terms contains one more variable of the remaining $(n - m)$ variables. Thus, if x_1, x_2 and x_3 are taken as repeating variables,

$$\pi_1 = x_1^{a_1} x_2^{b_1} x_3^{c_1} x_4$$

$$\pi_2 = x_1^{a_2} x_2^{b_2} x_3^{c_2} x_5$$

.

.

.

$$\pi_{n-m} = x_1^{a_{n-m}} x_2^{b_{n-m}} x_3^{c_{n-m}} x_n$$

Where $a_1, b_1, c_1; a_2, b_2, c_2$ etc. are constants to be determine as explained later.

While selecting m repeating variables, the following points should be kept in:

- m repeating variables must contain jointly all the fundamental dimensions involved in the phenomenon. Usually the fundamental dimensions are M, L and T. Therefore, 3 repeating variables must contain together M, L and T. However, if only two dimensions are involved, there will be 2 repeating variable and they must contain together the two dimensions involved.
- The repeating variables must not form the non-dimensional parameters amongst themselves.
- A geometrical property (such as length), a fluid property (such as mass density) and flow characteristics (such as velocity) are generally most suitable as repeating variables.

Example 2.3 Show that the frictional factor in an incompressible fluid flowing through pipe is expressed as

$$f = \phi \left[\frac{\rho V D}{\mu}, \frac{k}{D} \right]$$

Where

k = is height of roughness projection.

ϕ = represent a "function of"

Exercise 2.6 Show that a discharge over a spillway can be expressed as

$$Q = VD^2\phi\left[\frac{\sqrt{gD}}{V}, \frac{H}{D}\right]$$

Where,

V = velocity of flow,

D = throat depth,

H = head causing flow, and

g = acceleration due to gravity

Activity 2.4

What are the points should be considered while selecting repeating Variables?

SECTION-2: SIMILITUDE AND HYDRAULIC MODELS

Introduction

It is known that even with modern computing facilities, many complex problems still challenge complete theoretical analysis. A combination of past experience, theory and dimensional analysis will provide partial or complete solution to a number of problems. However, there still remain many problems, which are tractable only through experimentation. This will be done through model studies of proposed hydraulic structures and machines. So in this section we deal with the laws of similarity as applied to hydraulic structures.

In model analysis, investigations are made on a model which is similar to the full size structure known as proto type. Model testing is done to obtain useful quantitative or quantitative information that can be safely utilized in the design of the prototype.

A thorough knowledge of the principles of hydraulic similitude is essential in proper design, construction and operation of model.

2.2 SIMILITUDE

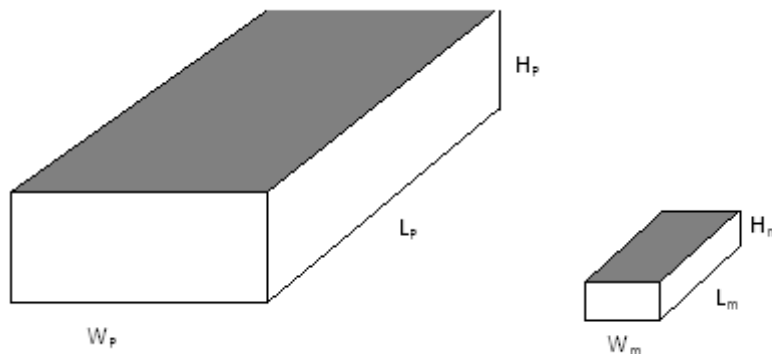
A model will yield useful quantitative information about the characteristics of the prototype if it is similar to its prototype. Complete similarity can be obtained between the model and its prototype if the two systems are geometrically, kinematically and dynamically similar.

Models which are completely similar with their prototype are known as TRUE MODELS. But if it is not possible to maintain geometric similarity between the model and the prototype the model is called DISTORTED MODEL.

TYPES OF SIMILARITY

(a) GEOMETRIC SIMILARITY

It is the similarity of the shape (scale factor). It is obtained when the solid boundaries that control the flow of fluid are geometrically similar. The model is a geometric reduction of the prototype and is accomplished by maintaining a fixed ratio of all homologous lengths between the model and prototype. These physical quantities are length, area, diameter, volume, etc.



Model scale ratio:

$$L_r = \frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{H_p}{H_m}$$

Area Ratio:

$$A_r = \frac{A_p}{A_m} = \frac{L_p}{L_m} * \frac{B_p}{B_m} = L_r^2$$

Volume ratio:

$$V_r = \frac{L_p}{L_m} * \frac{B_p}{B_m} * \frac{H_p}{H_m} = L_r^3$$

Activity 2.5

What is meant by geometric similarity?

(b) KINEMATIC SIMILARITY

It is the similarity of motion. For kinematic similarity to exist, the streamline pattern in the model must be the same as in its prototype. The ratios of kinematic quantities representing the flow characteristics such as, time, velocity, acceleration, and discharge must be the same at all corresponding points.

The velocity ratio is:

$$V_r = \frac{V_p}{V_m}$$

Time scale ratio:

$$T_r = \frac{T_p}{T_m} = \frac{L_r}{V_r} \quad (\text{because, } T = L/V)$$

Acceleration scale ratio:

$$a_r = \frac{a_p}{a_m} = \frac{L_r}{T_r^2} = \frac{V_r^2}{L_r} \quad (\text{because, } a = \left[\frac{L}{T^2} \right])$$

Discharge scale ratio:

$$Q_r = \frac{Q_r}{Q_m} = \frac{L_p^3 / T_p}{L_m^3 / T_m} = \frac{L_r^3}{T_r}$$

Activity 2.6

What is meant by kinematic similarity?

(c) DYNAMIC SIMILARITY

It is the Similarity of forces involved in motion. Dynamic Similarity is attained if the ratio of homologous forces in the model and prototype are kept constant.

$$i. e. \frac{F_p}{F_m} = F_r$$

The conditions required for complete similarity are developed from the Newton 2nd law of motion

$$i. e. \sum F_x = ma_x$$

The forces acting may be any one or a combination of several of the following: viscous, pressure, gravity, elasticity, surface tension, inertia forces etc.

$$\frac{\sum \text{Forces (viscous + pressure + gravity + elasticity + surface tension)}_p}{\sum \text{Forces (viscous + pressure + gravity + elasticity + surface tension)}_m} = \frac{m_p a_p}{m_m a_m}$$

$$\sum \vec{F} = \vec{F}_v + \vec{F}_g + \vec{F}_s + \vec{F}_p + \vec{F}_e = \text{RESULTANT} = -\vec{F}_i = m \cdot \vec{a}$$

NOTE: Newton's Law: Inertia force (F_i) is equal and opposite to the resultant forces.

$$\frac{(\sum F)_p}{(\sum F)_m} = \frac{(m \cdot a)_p}{(m \cdot a)_m} = \frac{(F_i)_p}{(F_i)_m} \quad (1)$$

However, in practice, a mode is designed to study the effects of only a few dominant forces. Dynamic similarity requires that the ratios of these forces be kept the same between the model and prototype.

$$\text{Inertia force, } F_i = m \cdot a = \rho \ell^3 \left(\frac{v^2}{\ell} \right) = \rho \ell^2 v^2$$

$$\text{Viscous force, } F_v = \tau A = \mu \frac{du}{dy} A = \mu \frac{v}{\ell} \cdot \ell^2 = \mu v \ell$$

$$\text{Gravity force, } F_g = m \cdot g = \rho \ell^3 g$$

$$\text{Pressure force, } F_p = p \cdot A = P \cdot \ell^2$$

$$\text{Elastic force, } F_e = E_v \cdot A = E_v \cdot \ell^2$$

$$\text{Surface tension force, } F_s = \sigma \ell$$

In problems of fluid flow, the inertia force will always exist and hence it is customary to find out the force ratios with respect to the inertia forces, thus:

Inertia to viscous forces ratio

$$\left(\frac{F_i}{F_v} \right)_m = \left(\frac{F_i}{F_v} \right)_p \quad (2)$$

Inertia to gravity forces ratio

$$\left(\frac{F_i}{F_g} \right)_m = \left(\frac{F_i}{F_g} \right)_p \quad (3)$$

Inertia to elastic forces ratio

$$\left(\frac{F_i}{F_e} \right)_m = \left(\frac{F_i}{F_e} \right)_p \quad (4)$$

Inertia to surface tension forces ratio

$$\left(\frac{F_i}{F_s} \right)_m = \left(\frac{F_i}{F_s} \right)_p \quad (5)$$

Inertia – to pressure forces ratio

$$\left(\frac{F_i}{F_p}\right)_m = \left(\frac{F_i}{F_p}\right)_p \quad (6)$$

The above six equations are dimensionless groups. The significance of the dimensionless ratios is discussed below:

Activity 2.7

What is meant by dynamic similarity?

(a) REYNOLDS NUMBER (*phenomenon governed by viscous force*)

A fluid in motion always involves inertia forces. If the inertial forces and viscous forces (example pipe flow) can be considered to be the only forces that govern the motion, the ratio of these forces acting on homologous particles in a model and its prototype is defined by the Reynolds number.

$$R_e = \frac{F_i}{F_v} = \frac{(\text{Inertial force})}{(\text{Viscous force})}$$

$$R_e = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho L V}{\mu} = \frac{L V}{\nu} \quad (\text{Non dimensional ratio})$$

The Reynolds number is important when the viscous force is predominant. It is generally used to correlate meter coefficient, pipe friction coefficient and drag coefficient, etc.

For dynamically similitude model and prototype:

$$(R_e)_m = (R_e)_p = \left(\frac{LV}{\nu}\right)_m = \left(\frac{LV}{\nu}\right)_p \quad \text{NOTE: } D \text{ is taken as } L \text{ for pipe flow.}$$

(b) FROUDE NUMBER: (*phenomenon governed by gravity force*)

When inertial forces and gravity forces are considered to be the only dominant forces in the fluid motion, the ratio of inertia forces to gravity forces acting on the homologous elements of the fluid in the model and prototype are considered as follows:

$$F_r = \frac{F_i}{F_g} = \frac{(\text{Inertial force})}{(\text{gravitational force})}$$

$$F_r = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho L^2 V^2}{\rho g L^3}} = \sqrt{\frac{V^2}{gL}}$$

The Froude number is important in open channel flow. It is useful in study of hydraulic jump, in design of hydraulic structures and ships, etc.

For dynamically similitude model and prototype:

$$(F_r)_m = (F_r)_p = \left(\frac{V}{\sqrt{gL}} \right)_m = \left(\frac{V}{\sqrt{gL}} \right)_p$$

N.B For open channel case L is the depth of the channel.

For a ship, L is taken as the length at water line.

(c) WEBER NUMBER (phenomenon governed by surface tension)

The surface tension is a measure of energy level on the surface of a liquid body. The force is of primary importance in hydraulic engineering practice in the study of small surface waves or control of evaporation from a large body of water, such as water storage tank / reservoir.

In river and harbor models reduction of scale often leads to appreciable viscous and capillary effects in the shallow regions of flow. The depth of flow in such cases should be sufficiently large so that capillary effects are negligible.

The ratio of Inertia to Surface tension forces in prototype and model is:

$$\frac{F_i}{F_s} = \frac{\rho V^2 L^2}{\sigma L} = \frac{V^2}{\sigma / \rho L}$$

The square root of this dimensionless ratio is known as WEBER NUMBE (W_e):

$$W_e = \frac{V}{\sqrt{\sigma / \rho L}}$$

It is applied at the leading edge of a very thin sheet of liquid flowing over a surface. Like:

- ☞ Capillary movement of water in solids.
- ☞ Flow of liquid at a very small depth over a surface.
- ☞ Flow over weir at very small heads.
- ☞ Spray of liquid from the exit of discharging tube resulting in the formation of drops of liquids.

(d) MACH NUMBER (phenomenon governed by elastic forces)

The Mach number can be regarded as the ratio of inertia and elastic forces. Is important in compressible fluid flow problems at high velocities, such as high velocity flow in pipes or motion of high speed projectiles and missiles, also important in

- ☞ Aerodynamic testing
- ☞ Flow gases exceeding the velocity of sound.
- ☞ Water hammer problems. (Design of surge tanks).

The ratio of Inertia forces & elastic forces in prototype and model is:

$$\frac{F_i}{F_e} = \frac{\rho L^2 V^2}{E L^2} = \frac{V^2}{E/\rho}$$

The square root of this dimensionless number is known as

Mach number (M). Thus:

$$M = \frac{V}{\sqrt{E/\rho}} = \frac{V}{C}$$

This is for fluid velocity (or velocity of the body through a stationary fluid) to that of a sound wave in the same medium.

C is the sonic velocity (or celerity) in the given medium.

(e) EULER NUMBER (Phenomenon governed by pressure forces.)

The ratio of Inertia forces to pressure forces for both prototype and model is given by:

$$\frac{F_i}{F_p} = \frac{\rho V^2 L^2}{PL^2} = \frac{V^2}{P/\rho}$$

The square root of this dimensionless number is known as EULER NUMBER (E_u). Thus:

$$E_u = \frac{V}{\sqrt{P/\rho}} = \frac{V}{\sqrt{2\Delta P/\rho}} = \frac{V}{\sqrt{2g(\Delta P/\gamma)}}$$

The Euler number is important in the flow problems in which a pressure gradient exists.

For complete dynamic similarity to exist between the model and the prototype and its model, all the above mentioned parameters should be satisfied. It is impossible to obtain complete dynamic similarity because no two fluids are known to have the requisite properties to satisfy all conditions. Fortunately, in most engineering problems, all conditions are not required to be satisfied because some of the forces (a) may not act, (b) may be insignificant, (c) may oppose each other and thus cancel out. Usually 2 or 3 types of forces may be predominant. A sound knowledge of the fluid phenomenon is essential to ascertain the forces that are significant in a particular phenomenon.

Example 2.4 The general expression for discharge of a centrifugal pump is

$$Q = ND^3 \phi \left[\frac{D}{B}, \frac{N^2 D^2}{gH}, \frac{\rho N D^2}{\mu} \right]$$

The performance of a new design of the pump is to be studied by testing a 1:6 *model*. the pump is to run at 1500 *r.p.m*

Example 2.5 The drag on a sphere is expressed as

$$F = \rho V^2 D^2 \phi \left(\frac{VD}{\nu} \right)$$

A Sphere of diameter D when placed in water moving with a velocity of 2 m/s experiences a drag of 6N. Determine the drag on another sphere of diameter 2D placed in a wind tunnel at the corresponding velocity. Take $\nu_a = 13\nu_w$, $\rho_a = 1.30 \text{ kg/m}^3$ and $\rho_w = 1000 \text{ kg/m}^3$

2.3 TYPES OF MODELS

In general hydraulic models can be classified under two broad categories

- (a) Undistorted models
- (b) Distorted models

(a) Undistorted Models: - if a model is geometrically similar to its prototype, it is known as undistorted models. i.e., the scale ratios for the corresponding linear dimension are the same.

(b) Distorted models: - if one or more terms of the models are not identical with the prototype it is known as distorted models. The distortion may be *geometrical, or material or hydraulic quantities or a combination of these*.

☞ **Geometrical distortion,**

The distortion can be either of dimensions or that of configuration.

- When different scale ratios are adopted for the longitudinal, transverse, & vertical dimensions; then it is known as distortion of dimensions.

It is adopted in river models where a different slope ratio for depth is adopted.

- Distortion of configuration results when the general configuration of the model doesn't have resemblance with its prototype.

If a river model has different bed slope ratio, this is distortion of configuration.

☞ **Material distortion** is occurred when the physical properties of the material used in the model and prototype are different.

☞ **Distortion of hydraulic quantities** is occurred for certain uncontrollable hydraulic quantities such as time, discharge etc.

The following are some of the reasons for adopting distorted models:

- a) To maintain accuracy in vertical measurements;
- b) To maintain turbulent flow;
- c) To obtain suitable bed material & its adequate movement;
- d) To obtain suitable roughness condition;

- e) To accommodate the available facilities such as space, money, water supply & time.

Activity 2.8

State and explain the two types of hydraulic models